Section 8.2

Integration by Parts

Like u-substitution giving the method of indefinite integration which undoes differentiation via the Chain Rule, we want to find a way to undo the Product Rule.
Let $f(x) = u(x)v(x)$ be the product of two differentiable functions. By the Product Rule, we know

$$f'(x) = u'(x)v(x) + u(x)v'(x)$$

or to save a bit of space

$$f' = u'v + uv'.$$

If we were to integrate $f'dx$, we would find

$$uv = f = \int f'\,dx = \int (u'v + uv')dx.$$

We can simply solve the above equation for $\displaystyle\int uv'\,dx$ to obtain an integration formula for the product

$$\int uv'\,dx = uv - \int vu'\,dx.$$

Like in our notational conventions for u-sub, we write $du = u'(x)dx$ and $dv = v'(x)dx$.

Theorem 8.2.1 (Integration by Parts)**.** If u and v are differentiable functions, then

$$\int u\,dv = uv - \int v\,du.$$

Example 8.2.2. Compute $\displaystyle\int xe^x\,dx$.

Reminder.

–You get to choose from the given integrand what will be u and what will be dv.

–Whatever you choose as dv, you **MUST** be able to recover v.

–The candidate for u should be considered in order of the acronym LIPET: **L**ogarithm, **I**nverse trig, **P**olynomial, **E**xponential, **T**rigonometric.

Example 8.2.3. Evaluate $\int_0^1 x^2 e^x \, dx$.

Example 8.2.4. Compute $\int \ln x \, dx$.

Example 8.2.5. Evaluate $\int e^x \sin x \, dx$.

Example 8.2.6. Compute $\int_1^{e^3} x^2 \ln x \, dx$.

Example 8.2.7. Compute $\int_0^1 \arcsin x \, dx$.